SMALL SIGNAL ANALYSIS OF HVDC-HVAC INTERACTIONS

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Abstract - This paper presents a small signal analysis of interactions between HVAC and HVDC systems. The eigenvalue analysis, along with analysis of participation factors, is firstly described. The eigenvalue sensitivity analysis shows how the changes of AC system parameters influence the stability of the considered HVDC-HVAC system. The influence of SCR changes on both the rectifier and the inverter AC system, is studied by examining the relative movement of the system eigenvalues. The most important conclusions about AC-DC interactions are obtained by investigating the nature of inherent feedback loops between the systems. By examining the changes of all interaction variables, it is determined which of the interaction variables should be controlled and which are better left uncontrolled.

Keywords: HVDC transmission, Power system dynamic stability, State space methods, Eigenvalues/eigenfunctions.

I. INTRODUCTION

The analysis presented in this paper, offers a qualitative insight into small signal HVDC-HVAC interaction mechanism in the frequency domain <100Hz, with emphasis on specific phenomena which have caused some of the experienced HVDC operating problems. Of major importance are the two groups of reported HVDC-HVAC interaction problems: the second harmonic instability (including the core saturation instability and periodic instabilities close to the first harmonic) and, problems related to the low Short Circuit Ratio (SCR) AC systems connected to DC systems.

The above presented operating problems have deserved a noticeable attention in the available HVDC bibliography. The second harmonic instability and core saturation instability are best described in [1] and [2]. Although the implications of the phenomenon were well elaborated and the solutions were found, an in-depth analytical explanation is lacking in these references. A recent paper [3] presents a new algorithm for analysis of the core saturation instability.

This algorithm, based on the experimentally calculated AC-DC interaction matrices, can accurately predict whether or not the instability will occur for a particular system and at a particular frequency. However, algorithms of this type, can not give qualitative conclusions about the mechanism and nature of the instability, and they do not examine which of the AC-DC interaction variables are mostly responsible for the instability.

The problems associated with low SCR AC systems are also well presented in literature [4],[5]. Similar to the second harmonic instability, an in-depth analytical explanation of the problem can not be found in the available literature.

The complexity of HVDC-HVAC interactions and well known difficulties in modelling of HVDC (HVDC-HVAC) systems, are the main reason for the non-availability of a wide analytical background to these problems.

The important points which need to be addressed in a study of HVDC-HVAC interactions can be summarised as:

- By knowing the independent behaviour of AC and DC systems, is it possible to predict the behaviour and possible stability problems when the systems are coupled together?
- What is the nature and the frequency range for the expected problems at a particular interconnection point? What side of the system (rectifier or inverter) is more likely to encounter operating difficulties?
- Since the SCR is a very important indicator of AC system characteristics, the analysis of eigenvalue movement as a consequence of SCR changes, would give invaluable practical information about expected stability problems.

The variables which characterise HVDC-HVAC interactions and which participate in inherent feedback loops between the systems, should be identified. Each one of these feedback loops and variables will naturally contribute either towards improvement or deterioration of the overall system stability. The frequency range where a particular variable mostly affects the system stability should also be determined. This analysis helps in determining the interaction variables which need to be controlled, and at the same time the interaction variables which are better to be left uncontrolled since they inherently improve the system stability.

This paper attempts to answer the above posed questions. The main method of the analysis is the eigenvalue decomposition.

II. SYSTEM MODEL
The system model used for the analysis presented in this paper, has been derived for the CIGRE HVDC Benchmark system (Rec $SCR=2.5$, Inv $SCR=2.5$) and the results presented in this paper relate to this test system. The model derivation, is described in detail in reference [6]. The model is linear continuous, and it includes dynamics of the DC system, the AC systems and the Phase Locked Loops (PLL) on both ends of HVDC link. All AC-DC interaction variables are conveniently represented to enable analysis of interactions between the subsystems.

The rectifier of the test system is in DC control mode whereas the inverter is in beta constant mode. The results and conclusions are very similar when the inverter is in gamma constant mode. Other operating modes, such as inverter in direct current control, do not occur very often under normal operating conditions and they have not been considered in this analysis.

III. EIGENVALUE DECOMPOSITION BASED ANALYSIS

This section analyses the sensitivity of dominant system eigenvalues, with respect to the system parameters. The aim of this analysis is to determine which subsystem is responsible for the movement of the each of the dominant eigenvalues. The frequency range of possible interaction problems as a consequence of change of operating conditions, can be readily obtained from this analysis.

Table 1 shows the 16 dominant eigenvalues of the test system (HVAC-HVDC-HVAC system). The frequency of the eigenvalues is shown as seen at the DC side, i.e. after $D-Q-0$ transformation is applied, in rad/sec. If the eigenvalues of independent subsystems (AC and DC systems) are analysed and compared with the eigenvalues of the overall system, it can be found that the eigenvalues 1-2 and 9-10 originate from the rectifier AC system and they have been significantly moved towards the imaginary axis, by the interconnection with DC system. Eigenvalues 7-8 originate from inverter AC system, and their movement towards imaginary axis is slower then the rectifier AC system eigenvalues. The eigenvalues 3-4 and 5-6 have been transformed from the rectifier/inverter AC system real eigenvalues. They did not move by the coupling with DC system, and for most of the considered system outputs, they appear with “pined” zeros, and thus have no importance in analysing the system behaviour. Hence, the real AC system poles do not seem to have any significance for AC-DC interactions. The eigenvalues 11-16 are coming from the PLL dynamics and from the low frequency DC line dynamics, and they do not undergo significant movement by joining of AC and DC systems.

The column five in Table 1 shows the results from the eigenvalue sensitivity analysis [7]. The partial derivative $\partial \lambda_i / \partial a_{ij}$, which indicates the sensitivity of eigenvalue to the changes in the system parameters, is calculated in this analysis. The considered parameters $[a_{ij}]$ are the elements of the system matrix $A$ in the state space system model. The four elements to which the considered eigenvalue is mostly sensitive are marked as “rec” or “inv” depending on whether they belong to rectifier AC system or to inverter AC system. Only the sensitivity to AC system parameters are considered, since it is known that AC system parameters change considerably with the change of operating conditions. Parameters of the DC systems, on the other hand, do not change during operation.

It is evident from these results that eigenvalues at higher frequency are more sensitive to rectifier AC system parameter changes. Even the eigenvalues originating from inverter AC system (eig. 7-8) are very sensitive to rectifier AC system parameters. The eigenvalues at lower frequency are far more sensitive to inverter AC system parameters.

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Damping ratio</th>
<th>Frequency [rad/sec]</th>
<th>Eigenvalue sensitivity</th>
<th>Participation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-25.257+ 444.46i</td>
<td>0.0567</td>
<td>445.2</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>2</td>
<td>-25.257- 444.46i</td>
<td>0.0567</td>
<td>445.2</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>3</td>
<td>-43.798+ 314.15i</td>
<td>0.1381</td>
<td>317.2</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>4</td>
<td>-43.798- 314.15i</td>
<td>0.1381</td>
<td>317.2</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>5</td>
<td>-43.783+ 314.15i</td>
<td>0.1380</td>
<td>317.2</td>
<td>Inv,Inv,Inv,Inv</td>
</tr>
<tr>
<td>6</td>
<td>-43.783- 314.15i</td>
<td>0.1380</td>
<td>317.2</td>
<td>Inv,Inv,Inv,Inv</td>
</tr>
<tr>
<td>7</td>
<td>-76.466+ 392.64i</td>
<td>0.1912</td>
<td>400.0</td>
<td>Rec,Rec,Inv,Rec</td>
</tr>
<tr>
<td>8</td>
<td>-76.466- 392.64i</td>
<td>0.1912</td>
<td>400.0</td>
<td>Rec,Rec,Inv,Rec</td>
</tr>
<tr>
<td>9</td>
<td>-124.22+ 206.43i</td>
<td>0.5156</td>
<td>240.9</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>10</td>
<td>-124.22- 206.43i</td>
<td>0.5156</td>
<td>240.9</td>
<td>Rec,Rec,Rec,Rec</td>
</tr>
<tr>
<td>11</td>
<td>-110.12</td>
<td></td>
<td></td>
<td>Inv,Inv,Inv,Rec</td>
</tr>
<tr>
<td>12</td>
<td>-53.116</td>
<td></td>
<td></td>
<td>Inv,Inv,Inv,Inv</td>
</tr>
<tr>
<td>13</td>
<td>-29.213</td>
<td></td>
<td></td>
<td>Rec,Rec,Rec,Inv</td>
</tr>
<tr>
<td>14</td>
<td>-12.069</td>
<td></td>
<td></td>
<td>Rec,Rec,Rec,Inv</td>
</tr>
<tr>
<td>15</td>
<td>-8.526</td>
<td></td>
<td></td>
<td>Inv,Inv,Rec,Rec</td>
</tr>
<tr>
<td>16</td>
<td>-7.0369</td>
<td></td>
<td></td>
<td>Inv,Inv,Inv,Inv</td>
</tr>
</tbody>
</table>

The column six shows the results from participation factor calculation. Participation factor indicates the degree of participation of each of the states in the considered eigenvalue. The indices of four states which participate mostly in the con-
sidered eigenvalue are shown in this column, where the states are grouped in the following manner [6]:

\[
\begin{align*}
  x_1 - x_{13} & \text{ DC system and PLL} \\
  x_{14} - x_{29} & \text{ Rectifier AC system} \\
  x_{30} - x_{45} & \text{ Inverter AC system}
\end{align*}
\]

The participation factor results are in agreement with eigenvalue sensitivity analysis.

As a conclusion from this analysis, it can be said that operating problems at rectifier side of HVDC link can be expected at higher frequencies, in the form of oscillatory (harmonic) instability. At inverter side the instabilities can occur at much lower frequencies. Note, the frequency ranges are defined in this paper as follows: lower frequency \(f<20\text{Hz}\), mid frequency \(20\text{Hz}<f<50\text{Hz}\), and higher frequency \(f>50\text{Hz}\), as referred to the DC side.

Therefore, the rectifier side AC system should be designed not to have resonant peaks at frequencies around the second harmonic, since this resonant peak can cause instabilities when AC and DC systems are connected. The same resonant peaks at inverter AC system are not likely to cause instabilities. An explanation for these conclusions lies in the HVDC control structure. It is known that DC current controller structure in an HVDC system is asymmetrical, with rectifier controller regulating current and inverter controller being passive. The fast acting controller on rectifier side will tend to excite “faster” eigenvalues from the connected AC system. Inverter AC system is connected to the passive end of HVDC link and it will tend to change the system time constants at lower frequencies, essentially contributing only to the “inertia” of the system.

IV. INFLUENCE OF AC SYSTEM SCR

The AC system SCR have traditionally been considered as the best indicator of possible AC-DC interaction problems. It is known that an HVDC system connected to a low SCR AC systems (SCR=2.5-3.5) may cause some operating problems. AC systems with very low SCRs (SCR<2.5 under normal operating conditions) are usually avoided.

This section uses the relative movement of the system eigenvalues to analyse the system behaviour under a reduced SCR. The SCR of each of the AC systems is reduced in small steps (keeping power factor constant) and the positions of the system eigenvalues are observed. Table 2 shows the dominant eigenvalues, for rectifier side SCR reduced to SCR=1.7, and for inverter side SCR reduced to the same value. It is evident from the table, that the reduced SCR affects predominantly the eigenvalues at lower frequencies. This relates to both, rectifier and inverter side reduced SCR, however, with some differences. Reduced SCR at inverter side causes rapid movement of newly created complex eigenvalues (eig. 12-13) towards the imaginary axis. The system becomes unstable for SCR close to 1.2. At SCR=1.9, the system responses were so distorted that a start up procedure was very difficult.

In the case of reduced SCR at rectifier side, the eigenvalues 11-12 (with much better damping) move towards imaginary axis with slower rate. At SCR=1.3, the system was operating with very little changes in the responses. The system was also still stable at SCR=0.7. As can be seen from Table 2, the damping of dominant oscillatory mode was even improved with reduced SCR at rectifier side. Figure 1 confirms that with \(SCR=1.43\) at rectifier side, the system response is not significantly deteriorated. The simulation results were obtained using PSCAD/EMTDC.

It is therefore evident that the system is very sensitive to reduced SCR at inverter side, and the instability can be expected at lower frequency. At the same time, quite significant reduction of SCR at rectifier side is not expected to induce much problems. This is an unfortunate fact, since it is known that the inverter side usually has lower SCR than the rectifier side.

### TABLE 2. SYSTEM EIGENVALUES FOR REDUCED SCR

<table>
<thead>
<tr>
<th>System</th>
<th>Original SCR=2.5</th>
<th>Rec. SCR=2.5, Inv. SCR=1.7</th>
<th>Rec. SCR=1.7, Inv. SCR=2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-25.25+444.46i</td>
<td>-18.14+447.69i</td>
<td>-37.19+409.76i</td>
</tr>
<tr>
<td>2</td>
<td>-25.25+444.46i</td>
<td>-18.14+447.69i</td>
<td>-37.19+409.76i</td>
</tr>
<tr>
<td>7</td>
<td>-76.46+392.64i</td>
<td>-43.04+359.95i</td>
<td>-58.06+386.15i</td>
</tr>
<tr>
<td>8</td>
<td>-76.46+392.64i</td>
<td>-43.04+359.95i</td>
<td>-58.06+386.15i</td>
</tr>
<tr>
<td>11</td>
<td>-110.12</td>
<td>-153.23014</td>
<td>-86.89+24.80i</td>
</tr>
<tr>
<td>12</td>
<td>-53.116</td>
<td>-12.78+23.68i</td>
<td>-86.89+24.80i</td>
</tr>
<tr>
<td>13</td>
<td>-29.213</td>
<td>-12.78-23.68i</td>
<td>-22.587</td>
</tr>
<tr>
<td>14</td>
<td>-12.069</td>
<td>-12.697</td>
<td>-11.532</td>
</tr>
<tr>
<td>15</td>
<td>-8.526</td>
<td>-8.207</td>
<td>-8.653</td>
</tr>
<tr>
<td>16</td>
<td>-7.0369</td>
<td>-6.566</td>
<td>-7.161</td>
</tr>
</tbody>
</table>

V. ANALYSIS OF INHERENT FEEDBACK LOOPS BETWEEN THE SYSTEMS

This section examines the influence of inherent feedback loops between AC and DC systems. As it can be readily identified, there are four AC-DC interaction variables: AC current magnitude, AC current angle, AC voltage magnitude and AC voltage angle. AC voltage magnitude and AC voltage angle are of special importance, because these variables are either controlled, or they can be controlled by some conventional means. Converter bus voltage magnitude is controlled by using some of the voltage controlling elements, whereas the AC voltage angle is “controlled” by converter PLL (more precisely, PLL does not control AC voltage angle, but it can shield the DC system from AC voltage angle disturbances).
This section attempts to determine the specific feedback loops that negatively influence the system stability. Once they are determined it is also meaningful to know how “bad” they are, and at what frequency range their influence is predominant.

Considering the interaction of AC and DC systems through any physical variable (interaction variable), its influence on the system performance will be reduced (or it could be completely eliminated) if the variable is controlled by some external means such that it is kept as close to its steady-state value as possible. The degree of system interaction through this variable can be evaluated by comparing the performance of such a controlled system against the natural, uncontrolled system.

Let the system state equations are described in its usual form as:

$$\dot{x} = Ax + Bu, \quad y = Cx$$

(1)

The system equations from the analytical model in use, are purposely formulated such that the previously described interaction variables are represented as state variables.

To simulate external control of these state (interaction) variables the following method is used: The considered state variable $x_i$ is replaced by a new variable $x_{new}$ in (1), where the new state variable is obtained as shown in Figure 2.

The controller in Figure 2 is intended to control the new variable at its reference value, ie. at $x_{new} = 0$. If the controller gain $k_c$ has very low values, then the new variable will have dynamics similar to that of the variable $x_i$. In the case of $k_c \to 0$ the two variables will be the same ($x_i = x_{new}$) and this system is referred here as the original system.

If the controller gain is increased, the new variable becomes faster controlled at its steady-state value and the influence of the original variable becomes attenuated. In the extreme case when $k_c \to \infty$, the new variable becomes suppressed at its nominal value ($x_{new} = 0$). In this case, the case of ideal control, the considered variable will not have influence on the dynamics of the rest of the system. The case of ideal control is equivalent to deleting the corresponding row and column from (1), ie. $x_i = 0$, $x_{new} = 0$.

By continuously varying the gain $k_c$, the influence of each of the interaction variable on the system stability can be examined.

Figure 3 shows the root locus when the gain $k_c$ is varied for each interaction variable independently. The general conclusions from Figure 3 with respect to inherent AC-DC interaction loops, are different for rectifier and for inverter side, and can be summarised as follows:

**Rectifier side**

From figure 3 a), it can be observed that AC voltage magnitude has a significant influence on the system stability, and controlling of this variable can considerably change the system behaviour. A tight control of this variable, especially at mid-lower frequencies, would have deteriorating affect on the system stability, as shown by the ends of the root locus branches. The system behaviour in the higher frequency range is of less importance, since it is known that with practically available voltage controlling elements no effective control action is imposed on AC voltage at such high frequencies.
For the case of weaker control of AC voltage magnitude, the system stability can be improved in the whole frequency range. It is therefore difficult to strictly label the influence of this variable as positive or negative. In overall, it can be seen that the system stability can be improved by controlling the AC voltage magnitude, but careful tuning of controller gains is necessary.

As can be seen in Figure 3 b), the control of AC voltage angle would improve the system stability in the mid and higher frequency range. Some negative influence is however introduced at lower frequencies. Since the AC-DC interactions at rectifier side at higher frequencies are of more importance, this loop can be regarded as negative, and it should be externally controlled. It is important that the damping of dominant oscillatory mode can also be improved by the AC voltage angle control. The inherent negative influence of this loop can be readily eliminated by increasing rectifier side PLL gains. Simulation results, shown in Figure 4, confirm this conclu-
Inverter side

Figure 3 c) shows that an ideal control of the AC voltage magnitude at inverter side would deteriorate the system stability in the whole frequency range. However with relatively low controller gains, the system stability could be improved in the mid-lower frequency range.

The potential control of AC voltage angle, as can be seen from Figure 3 d), would substantially degrade the system stability in the whole frequency range. The most pronounced negative influence is at lower frequencies. It is evident that the AC voltage angle at inverter side represents inherent negative feedback loop for the considered system, and if this variable is externally controlled the system behaviour becomes worsen. Therefore the PLL gains at inverter side should be tuned to low values. Figure 5 shows the simulated system response with inverter side PLL gains increased ten times. As can be seen, a new low-frequency oscillatory mode (≅15Hz) is looming and the stability is noticeably deteriorated.

The PLL gains at inverter side should be kept at lower values especially in the case of low SCR AC system, for it was shown in section IV that the reduced SCR predominantly affects the eigenvalues at lower frequencies. Traditionally, it was a common practice to tune both: rectifier and inverter PLL gains, at very low values in order to maintain the system stability [8]. However as it was shown earlier, the rectifier side PLL gains could be kept at far larger values.

The analysis presented also provides an insight for the possible design of improved PLL controllers. By the selective control of AC voltage angle throughout the considered frequency range a purposely designed PLL controller with cascade compensator(s) could further improve the system stability.

VI. CONCLUSIONS

A small signal analysis of HVDC-HVAC systems is presented in this paper. The eigenvalue sensitivity and participation factor analysis have been used for the identification of frequency range for possible AC-DC interaction problems. The variations in rectifier side AC system parameters can cause oscillatory instabilities with frequencies around first harmonic. The variations in inverter AC system parameters are likely to cause instability at a much lower frequency.

The analysis of influence of SCR changes revealed that the system is very sensitive to SCR reduction at inverter side. At the same time, an excellent robustness to SCR reduction at rectifier side was noticed.

The examination of inherent feedback loops between the subsystems has shown that AC voltage angle changes at rectifier side, can deteriorate the system stability if left uncontrolled. Consequently, the rectifier side PLL gains should be tuned to higher values. On the other hand, AC voltage magnitude should not be tightly controlled at rectifier side.

At inverter side, the control of AC voltage angle deteriorates the system stability. This requires the PLL gains to be kept at lower values. Some stability improvement is possible by external control of AC voltage magnitude.

REFERENCES:


BIographies

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