An axiomatic approach to support in argumentation

Claudette Cayrol and Marie-Christine Lagasquie-Schiex

IRIT-UPS, Toulouse, France
{ccayrol,lagasq}@irit.fr

Abstract. In the context of bipolar argumentation (argumentation with two kinds of interaction, attacks and supports), we present an axiomatic approach for taking into account a special interpretation of the support relation, the necessary support. We propose constraints that should be imposed to a bipolar argumentation system using this interpretation. Some of these constraints concern the new attack relations, others concern acceptability. We extend basic Dung's framework in different ways in order to propose frameworks suitable for encoding these constraints. By the way, we propose a formal study of properties of necessary support.

Keywords: abstract argumentation, bipolar argumentation, axiomatization of necessary support

1 Introduction

The main feature of argumentation framework is the ability to deal with incomplete and / or contradictory information, especially for reasoning [15; 2]. Moreover, argumentation can be used to formalize dialogues between several agents by modeling the exchange of arguments in, *e.g.*, negotiation between agents [4]. An argumentation system (AS) consists of a collection of arguments interacting with each other through a relation reflecting conflicts between them, called *attack*. The issue of argumentation is then to determine "acceptable" sets of arguments (*i.e.*, sets able to defend themselves collectively while avoiding internal attacks), called *"extensions"*, and thus to reach a coherent conclusion. Formal frameworks have greatly eased the modeling and study of AS. In particular, the framework of [15] allows for abstracting the "concrete" meaning of the arguments and relies only on binary interactions that may exist between them.

In this paper, we are interested in bipolar AS (BAS), which handle a second kind of interaction, the support relation. This relation represents a positive interaction between arguments and has been first introduced by [18; 27]. In [8], the support relation is left general so that the bipolar framework keeps a high level of abstraction. However there is no single interpretation of the support, and a number of researchers proposed specialized variants of the support relation: deductive support [5], necessary support [21; 22], evidential support [23; 24], backing support [13]. Each specialization can be associated with an appropriate modelling using an appropriate complex attack. These proposals have been developed quite independently, based on different intuitions and with different formalizations. [10] presents a comparative study in order to restate these proposals in a common setting, the bipolar argumentation framework (see also [13] for another survey). The idea is to keep the original arguments, to add complex attacks defined by the combination of the original attack and the support, and to modify the classical notions of acceptability. An important result of [10] is the highlight of a kind of duality between the deductive and the necessary specialization of support, which results in a duality in the modelling by complex attacks. In this context, new different papers have recently been written: some of them give a translation between necessary supports and evidential supports [25]; others propose a justification of the necessary support using the notion of subarguments [26]; an extension of the necessary support is presented in [20]. From all these works it seems interesting to focus on the necessary support. However, different interpretations remain possible, leading to different ways of introducing new attacks and different ways to define acceptability of sets of arguments.

Our purpose is to propose a kind of "axiomatic approach" for studying how necessary support should be taken into account. Indeed we propose requirements (or constraints) that should be imposed to a bipolar argumentation system as "axioms" describing a desired behaviour of this system. Some of these constraints concern the new attack relations, others concern acceptability. We extend basic Dung's framework in different ways in order to propose frameworks suitable for encoding these contraints. By the way, we propose a formal study of properties of necessary support.

Some background is given in Section 2 for AS and BAS, in particular the duality identified in [10]. Section 3 presents constraints that should be imposed for taking into account necessary support. Then different frameworks for handling these constraints are described in Section 4. Section 5 concludes and suggests perspectives of our work. The proofs are given in [11].

2 Background on abstract bipolar argumentation systems

Bipolar abstract argumentation systems extend Dung's argumentation systems. So first we recall Dung's framework for abstract argumentation systems.

2.1 Dung's framework

Dung's abstract framework consists of a set of arguments and only one type of interaction between them, namely attack. The important point is the way arguments are in conflict.

Def. 1 (Dung AS) *A Dung's argumentation system (AS, for short) is a pair* $\langle \mathbf{A}, \mathbf{R} \rangle$ *where* $\mathbf{A}$ *is a finite and non-empty set of arguments and* $\mathbf{R}$ *is a binary relation over* $\mathbf{A}$ *(a subset of* $\mathbf{A} \times \mathbf{A}$*), called the* attack relation.

An argumentation system can be represented by a directed graph, called the *interaction graph*, in which nodes represent arguments and edges are defined by the attack relation: $\forall a, b \in \mathbf{A}$, $a\mathbf{R}b$ is represented by $a \not\to b$.

Def. 2 (Admissibility in AS) *Given* $\langle \mathbf{A}, \mathbf{R} \rangle$ *and* $S \subseteq \mathbf{A}$. S *is* conflict-free *in* $\langle \mathbf{A}, \mathbf{R} \rangle$ *if and only if (iff, for short) there are no arguments* $a, b \in S$*, such that (s.t., for short)* $a\mathbf{R}b$. $a \in \mathbf{A}$ *is* acceptable *in* $\langle \mathbf{A}, \mathbf{R} \rangle$ *with respect to (wrt, for short)* S *iff* $\forall b \in \mathbf{A}$ *s.t.* $b\mathbf{R}a$, $\exists c \in S$ *s.t.* $c\mathbf{R}b$. S *is* admissible *in* $\langle \mathbf{A}, \mathbf{R} \rangle$ *iff* S *is conflict-free and each argument in* S *is acceptable wrt* S.

Standard semantics introduced by Dung (preferred, stable, grounded) enable to characterize admissible sets of arguments that satisfy some form of optimality.

Def. 3 (Extensions) *Given* $\langle \mathbf{A}, \mathbf{R} \rangle$ *and* $S \subseteq \mathbf{A}$. S *is* a preferred extension *of* $\langle \mathbf{A}, \mathbf{R} \rangle$ *iff it is a maximal (wrt* $\subseteq$*) admissible set.* S *is* a stable extension *of* $\langle \mathbf{A}, \mathbf{R} \rangle$ *iff it is conflict-free and for each* $a \notin S$*, there is* $b \in S$ *s.t.* $b\mathbf{R}a$. S *is* the grounded extension *of* $\langle \mathbf{A}, \mathbf{R} \rangle$ *iff it is the least (wrt* $\subseteq$*) admissible set* X *s.t. each argument acceptable wrt* X *belongs to* X.

Ex. 1 *Let* AS *be defined by* $\mathbf{A} = \{a, b, c, d, e\}$ *and* $\mathbf{R} = \{(a, b), (b, a), (b, c), (c, d), (d, e), (e, c)\}$. *There are two preferred extensions (*$\{a\}$ *and* $\{b, d\}$*), one stable extension (*$\{b, d\}$*) and the grounded extension is the empty set.*

2.2 Abstract bipolar argumentation systems

The abstract bipolar argumentation framework presented in [8; 9] extends Dung's framework in order to take into account both negative interactions expressed by the attack relation and positive interactions expressed by a support relation (see [3] for a more general survey about bipolarity in argumentation).

Def. 4 (BAS) *A bipolar argumentation system (BAS, for short) is a tuple* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *where* $\mathbf{A}$ *is a finite and non-empty set of arguments,* $\mathbf{R}_{\text{att}}$ *is a binary relation over* $\mathbf{A}$ *called the* attack relation *and* $\mathbf{R}_{\text{sup}}$ *is a binary relation over* $\mathbf{A}$ *called the* support relation.

A BAS can still be represented by a directed graph[1], called the *bipolar interaction graph*, with two kinds of edges. Let a_i and $a_j \in \mathbf{A}$, $a_i\mathbf{R}_{\text{att}}a_j$ (resp. $a_i\mathbf{R}_{\text{sup}}a_j$) means that a_i attacks a_j (resp. a_i supports a_j) and it is represented by $a \not\to b$ (resp. $a \to b$).

Handling support and attack at an abstract level has the advantage to keep genericity. An abstract bipolar framework is useful as an analytic tool for studying different notions of complex attacks, complex conflicts, and new semantics taking into account both kinds of interactions between arguments. However, the drawback is the lack of guidelines for choosing the appropriate definitions and semantics depending on the application. For solving this problem, some specializations of the support relation have been proposed and discussed recently. The distinction between deductive and necessary support has appeared first. Then, several interpretations have been given to the necessary support (sub-argument relation [26], evidential support [23; 24; 25], backing support [13]).

Deductive support The deductive support has first appeared in [5]. This variant is intended to enforce the following constraint: If $b\mathbf{R}_{\text{sup}}c$ then "the acceptance of b implies the acceptance of c", and as a consequence "the non-acceptance of c implies the non-acceptance of b".

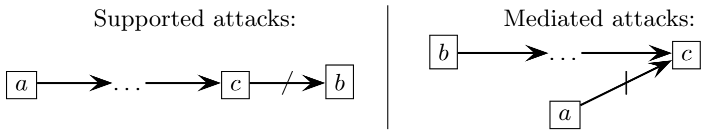
In relevant literature, this interpretation is usually taken into account by adding two kinds of complex attack. The idea is to produce a new AS, containing original and new attacks, and then to use standard semantics.

[1] This is an abuse of language since, stricly speaking, this is an edge-labeled graph (with two labels) rather than a directed graph.

The first new attack, called mediated attack in [5], occurs when $b\mathbf{R}_{\text{sup}}c$ and $a\mathbf{R}_{\text{att}}c$: "the acceptance of a implies the non-acceptance of c" and so "the acceptance of a implies the non-acceptance of b". Another complex attack, called supported attacks in [9] occurs when $a\mathbf{R}_{\text{sup}}c$ and $c\mathbf{R}_{\text{att}}b$: "the acceptance of a implies the acceptance of c" and "the acceptance of c implies the non-acceptance of b"; so, "the acceptance of a implies the non-acceptance of b".

Def. 5 ([5] Mediated attack, [9] Supported attack)
Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$*. There is a* mediated attack *from a to b iff there is a sequence* $a_1\mathbf{R}_{\text{sup}} \dots \mathbf{R}_{\text{sup}}a_{n-1}$*, and* $a_n\mathbf{R}_{\text{att}}a_{n-1}$*,* $n \geq 3$*, with* $a_1 = b$*,* $a_n = a$*. There is a* supported attack *from a to b iff there is a sequence* $a_1\mathbf{R}_1 \dots \mathbf{R}_{n-1}a_n$*,* $n \geq 3$*, with* $a_1 = a$*,* $a_n = b$*,* $\forall i = 1 \dots n-2$*,* $\mathbf{R}_i = \mathbf{R}_{\text{sup}}$ *and* $\mathbf{R}_{n-1} = \mathbf{R}_{\text{att}}$*.*

So, with the deductive interpretation of the support, new kinds of attack, from a to b, can be considered in the following cases:



Necessary support The necessary support has been first proposed by [21; 22] with the following interpretation: If $c\mathbf{R}_{\text{sup}}b$ then "the acceptance of c is necessary to get the acceptance of b", or equivalently "the acceptance of b implies the acceptance of c". A example of this kind of support could be:

Ex. 2 *A dialog between three customers about the qualities of services of their hotel:*
- *"This hotel is very well managed." (Argument a)*
- *"Yes. In particular, the hotel staff is very competent." (Argument b)*
- *"They are not competent! The rooms are dirty." (Argument c)*

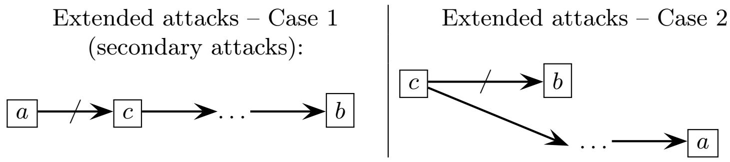
Here b necessarily supports a and c attacks b ($c \not\to b \to a$)*. The link between b and a is similar to the notion of subargument used in [26].*

As for deductive support, the idea is to add complex attacks in order to use standard semantics on a new AS. The first added complex attack, called extended attack in [21] and secondary attack in [9] has been proposed in the following case: Suppose that $a\mathbf{R}_{\text{att}}c$ and $c\mathbf{R}_{\text{sup}}b$. "The acceptance of a implies the non-acceptance of c" and so "the acceptance of a implies the non-acceptance of b". Another kind of complex attack may be considered when $c\mathbf{R}_{\text{sup}}a$ and $c\mathbf{R}_{\text{att}}b$: "the acceptance of a implies the acceptance of c" and "the acceptance of c implies the non-acceptance of b". So, "the acceptance of a implies the non-acceptance of b". This new attack from a to b has been proposed in [22].

The formal definition of these two attacks is:

Def. 6 ([22] Extended attack) *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$*. There is an* extended attack *from a to b iff either* $a\mathbf{R}_{\text{att}}b$ *(direct attack), or there is a sequence* $a_1\mathbf{R}_{\text{att}}a_2\mathbf{R}_{\text{sup}} \dots \mathbf{R}_{\text{sup}}a_n$*,* $n \geq 3$*, with* $a_1 = a$*,* $a_n = b$ *(Case 1), or there is a sequence* $a_1\mathbf{R}_{\text{sup}} \dots \mathbf{R}_{\text{sup}}a_n$*, and* $a_1\mathbf{R}_{\text{att}}a_p$*,* $n \geq 2$*, with* $a_n = a$*,* $a_p = b$ *(Case 2).*

So, with the necessary interpretation of the support, new kinds of attack, from a to b, can be considered in the following cases:



Duality between deductive and necessary support Deductive support and necessary support have been introduced independently. Nevertheless, they correspond to dual interpretations of the notion of support. Let us denote $a \stackrel{D}{\to} b$ (resp. $a \stackrel{N}{\to} b$) when there exists a deductive (resp. necessary) support from a to b. As $a \stackrel{D}{\to} b$ means that "the acceptance of a implies the acceptance of b", and $a \stackrel{N}{\to} b$ means that "the acceptance of a is necessary to get the acceptance of b", it follows that $a \stackrel{N}{\to} b$ is equivalent to $b \stackrel{D}{\to} a$.

Following this duality, it is easy to see that the mediated attack obtained by combining the attack relation $\mathbf{R}_{\text{att}}$ and the support relation $\mathbf{R}_{\text{sup}}$ exactly corresponds to the secondary attack obtained by combining the attack relation $\mathbf{R}_{\text{att}}$ and the support relation $\mathbf{R}_{\text{sup}}^{-1}$ which is the symmetric relation of $\mathbf{R}_{\text{sup}}$ ($\mathbf{R}_{\text{sup}}^{-1} = \{(b, a)|(a, b) \in \mathbf{R}_{\text{sup}}\}$). Similarly, the supported attack obtained by combining the attack relation $\mathbf{R}_{\text{att}}$ and the support relation $\mathbf{R}_{\text{sup}}$ exactly corresponds to the second case of extended attack obtained by combining the attack relation $\mathbf{R}_{\text{att}}$ and the support relation $\mathbf{R}_{\text{sup}}^{-1}$.

So in the following, we only focus on the necessary support since, taking advantage of the duality, all the results we obtain can be easily translated into results for deductive supports.

3 Axiomatic approach for handling necessary support

In relevant literature, as described in the previous section, taking into account support generally leads to add new attacks. It is the case for instance with the necessary support that leads to extended attacks. However, a deeper analysis of the original interpretation of necessary support suggests other ways to handle this support. In this section, we discuss several constraints induced by the intended meaning of necessary support, and we show that new frameworks must be proposed for encoding these constraints.

Let us come back to the original interpretation of necessary support: If $c\mathbf{R}_{\text{sup}}b$, "the acceptance of c is necessary to get the acceptance of b". Analysing this interpretation leads to at least four kinds of constraints.

Transitivity (TRA) This first requirement concerns the relation $\mathbf{R}_{\text{sup}}$ alone. It expresses transitivity[2] of the necessary support. It induces that a sequence of supports is considered as a support:

Def. 7 (Constraint TRA) *$\forall a, b \in \mathbf{A}$, if $\exists n > 1$ such that $a = a_1\mathbf{R}_{\text{sup}} \ldots \mathbf{R}_{\text{sup}}a_n = b$, then a* `supports` *b.*

[2] Irreflexivity has also been considered for instance in [21; 22].

Closure (CLO) A second constraint also concerns the relation $\mathbf{R}_{\text{sup}}$ alone and expresses the fact that if $c\mathbf{R}_{\text{sup}}b$, then "the acceptance of b implies the acceptance of c". So, if $c\mathbf{R}_{\text{sup}}b$, and there exists an extension S containing b, then S also contains c. This constraint can be expressed by the property of closure of an extension under $\mathbf{R}_{\text{sup}}^{-1}$:[3]

Def. 8 (Constraint CLO) *Let s be a semantics and E be an extension under s. $\forall a, b \in \mathbf{A}$, if $a\mathbf{R}_{\text{sup}}b$ and $b \in E$, then $a \in E$.*

Moreover, an interesting variant of this constraint could be induced by a slightly different reading of the original interpretation: "the acceptance of c is necessary to get the acceptance of b" because *c is the only attacker of a particular attacker of b*. This reading implies that there implicitly exists a special attack to b which can be only defeated by c. This interpretation will lead us to propose a framework with meta-arguments (see Section 4.2).

Conflicting sets (CFS) Now, we consider constraints induced by the presence of both attacks and supports in a BAS. Starting from the original interpretation, if $a\mathbf{R}_{\text{att}}c$ and $c\mathbf{R}_{\text{sup}}b$, "the acceptance of a implies the non-acceptance of c" and "the acceptance of b implies the acceptance of c". So, using contrapositives, "the acceptance of a implies the non-acceptance of b", and then "the acceptance of b implies the non-acceptance of a". Thus, we obtain a symmetric constraint involving a and b. However, the fact that "the acceptance of a implies the non-acceptance of b" is not equivalent to the fact that there is an attack from a to b. We have only the sufficient condition. So, the creation of a complex attack (here a secondary attack) from a to b can be viewed in some sense too strong. Hence, faced with the case when $a\mathbf{R}_{\text{att}}c$ and $c\mathbf{R}_{\text{sup}}b$, we propose to assert a conflict between a and b, or in other words that the set $\{a, b\}$ is a conflicting set. Similarly, if $c\mathbf{R}_{\text{att}}b$ and $c\mathbf{R}_{\text{sup}}a$, "the acceptance of a implies the acceptance of c" and so "the acceptance of a implies the non-acceptance of b".

Def. 9 (Constraint CFS) *$\forall a, b, c \in \mathbf{A}$. If ($a\mathbf{R}_{\text{att}}c$ and c `supports` b) or ($c\mathbf{R}_{\text{att}}b$ and c `supports` a) then $\{a, b\}$ is a conflicting set.*

Note that the Dung's abstract framework is not suitable for expressing such a constraint. So we will present in Section 4.1 a new framework for handling conflicting sets of arguments.

Addition of new attacks (nATT and n+ATT) Beyond these properties, according to the applications and the previous works presented in literature, we may impose stronger constraints corresponding to the addition of new attacks. Two cases may be considered:

Def. 10 (Constraint nATT) *If $a\mathbf{R}_{\text{att}}c$ and $c\mathbf{R}_{\text{sup}}b$, then there is a new attack from a to b.*

Def. 11 (Constraint n+ATT) *If ($a\mathbf{R}_{\text{att}}c$ and $c\mathbf{R}_{\text{sup}}b$) or ($c\mathbf{R}_{\text{att}}b$ and $c\mathbf{R}_{\text{sup}}a$), then there is a new attack from a to b.*

[3] Note that if $c\mathbf{R}_{\text{sup}}b$ and $c\mathbf{R}_{\text{att}}b$, as an extension must be conflict-free, there is no extension containing both c and b, so the constraint trivially holds. Some works, as for instance [10], exclude the case when $c\mathbf{R}_{\text{sup}}b$ and $c\mathbf{R}_{\text{att}}b$.

nATT (resp. **n+ATT**) corresponds to the addition of secondary (resp. extended) attacks. In Section 4.3 we present two frameworks for handling these constraints.

Continuing the discussion one step further, if the fact that "the acceptance of a implies the non-acceptance of b" is represented by an attack from a to b, due to contrapositive, this new attack must be symmetric. However, in that case, each attack should be turned into a symmetric one. Thus, we move towards symmetric argumentation frameworks which have been studied in [14]. We will not consider this case in the current paper. Some of the above constraints can he handled in a Dung's abstract framework (**CLO**, **TRA**, **nATT** and **n+ATT**) with the advantage of reusing all known Dung's results. However, as we noticed above, constraint **CFS** cannot be encoded in a Dung's framework. So in the next section we propose different variants of Dung's framework and of the bipolar framework in order to take into account these constraints.

4 New frameworks for handling necessary supports

Starting from the constraints discussed in Section 3, we propose several frameworks for handling necessary support. The first two are driven by Constraint **CLO** whereas the last two are driven by the constraints **nATT** and **n+ATT**. The section will end by a comparison of these frameworks.

4.1 Handling conflicting sets of arguments

We propose a generalized bipolar abstract argumentation framework consisting of a set of arguments, a binary relation representing an attack between arguments, a binary relation representing a support between arguments and a set of conflicting sets of arguments. Intuitively, knowing that a attacks b is stronger than knowing that $\{a, b\}$ is a conflicting set of arguments. Knowing that a set of arguments S is conflicting will only prevent any extension from containing S. Moreover, a conflicting set may contain more than two arguments.

Def. 12 (Generalized BAS, GBAS) *A generalized bipolar argumentation system is a tuple* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, \mathbf{C} \rangle$ *where* $\mathbf{A}$ *is a finite and non-empty set of arguments,* $\mathbf{R}_{\text{att}}$ *is a binary relation over* $\mathbf{A}$ *called the* attack relation*,* $\mathbf{R}_{\text{sup}}$ *is a binary relation over* $\mathbf{A}$ *called the* support relation *and* $\mathbf{C}$ *is a finite set of subsets of* $\mathbf{A}$ *such that* $\forall (a, b) \in \mathbf{R}_{\text{att}}, \{a, b\} \in \mathbf{C}$.

Conflict-freeness in a generalized bipolar argumentation system is defined as follows:

Def. 13 (Conflict-freeness in a GBAS) *Let* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, \mathbf{C} \rangle$ *be a GBAS and* $S \subseteq \mathbf{A}$*.* S *is conflict-free in the GBAS iff there does not exist* $C \in \mathbf{C}$ *such that* $C \subseteq S$*.*

However, the definition of semantics depends on the interpretation of the support and also on the constraints that have to be enforced. The generalized bipolar framework can be instantiated for encoding necessary support, due to the following definition:

Def. 14 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *with* $\mathbf{R}_{\text{sup}}$ *being a set of necessary supports. The tuple* $\mathsf{GBAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, \mathbf{C} \rangle$ *with* $\mathbf{C} = \{\{a,b\} | (a,b) \in \mathbf{R}_{\text{att}}\} \cup \{\{a,b\} | a\mathbf{R}_{\text{att}}c$ *and* c `supports` $b\} \cup \{\{a,b\} | c\mathbf{R}_{\text{att}}b$ *and* c `supports` $a\}$ *is the generalized argumentation system associated with* BAS.

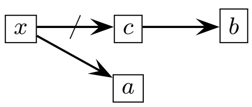
It is easy to see that the generalized argumentation system associated with BAS enables to enforce the constraints **TRA** and **CFS**, whereas it satisfies neither Constraint **nATT**, nor Constraint **n+ATT**.

The next step is the study of acceptability in a GBAS in order to check whether Contraint **CLO** is taken into account. For that purpose, the first proposal is to use conflict-freeness as defined in Def. 13 and admissible, preferred and stable extensions as defined in Dung's systems. In this case, it can be proved that every stable extension is closed under $\mathbf{R}_{\text{sup}}^{-1}$.

Prop. 1 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and its associated* GBAS. *Let* $S \subseteq \mathbf{A}$. *If* S *is conflict-free in* GBAS, *and for each* $a \notin S$, *there is* $b \in S$ *s.t.* $b\mathbf{R}_{\text{att}}a$, *then* S *is closed under* $\mathbf{R}_{\text{sup}}^{-1}$.

However, this approach produces many conflicts, without adding any attacks. So in many cases, there will be no stable extension. Moreover, Constraint **CLO** is generally not satisfied with the preferred semantics. The following example illustrates these two drawbacks.

Ex. 3 *Consider* BAS *represented by the following graph.*



$\mathbf{C} = \{\{x,c\},\{x,b\},\{a,c\}\}$. *Using the classical definition of semantics with conflict-freeness as defined in Def. 13, the preferred extensions of the associated* GBAS *are* $\{a,x\}$ *and* $\{a,b\}$, *and there is no stable extension. Moreover, the preferred extension* $\{a,b\}$ *is not closed under* $\mathbf{R}_{\text{sup}}^{-1}$.

The preferred semantics has to be redefined in order to enforce Constraint **CLO**. So, our second proposal is to enforce a notion of coherence by combining conflict-freeness and closure under $\mathbf{R}_{\text{sup}}^{-1}$. Moreover it can be proven that:

Prop. 2 *Let* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and its associated* GBAS. *Let* $S \subseteq \mathbf{A}$. *If* S *is closed under* $\mathbf{R}_{\text{sup}}^{-1}$ *then (*S *is conflict-free in* GBAS *iff* S *is conflict-free in* $\langle \mathbf{A}, \mathbf{R}_{\text{att}} \rangle$*).*

Def. 15 (Coherence in a GBAS) *Let* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, \mathbf{C} \rangle$ *be a GBAS and* $S \subseteq \mathbf{A}$. S *is coherent in the GBAS iff* S *is conflict-free in* $\langle \mathbf{A}, \mathbf{R}_{\text{att}} \rangle$ *and* S *is closed under* $\mathbf{R}_{\text{sup}}^{-1}$.

Using coherence in place of conflict-freeness leads to new definitions:

Def. 16 (Admissibility in a GBAS) *Let* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, \mathbf{C} \rangle$ *be a GBAS and* $S \subseteq \mathbf{A}$. S *is* admissible in the GBAS *iff* S *is coherent in the GBAS and* $\forall a \in S$, $\forall b \in \mathbf{A}$ *s.t.* $b\mathbf{R}_{\text{att}}a$, $\exists c \in S$ *s.t.* $c\mathbf{R}_{\text{att}}b$. S *is* a preferred extension of the GBAS *iff it is a maximal (wrt* $\subseteq$*) admissible set.* S *is* a stable extension of the GBAS *iff* S *is coherent*[4] *in the GBAS and for each* $a \notin S$, *there is* $b \in S$ *s.t.* $b\mathbf{R}_{\text{att}}a$.

[4] Due to Prop.1, coherent may be replaced by conflict-free.

Ex.3 (cont'd) *Taking into account coherence, as in Def.16,* $\{a, x\}$ *is the unique preferred extension of the associated* GBAS*, and it is closed under* $\mathbf{R}_{\text{sup}}^{-1}$.

So, using Def.16 and 15, the associated GBAS enables to enforce Constraint **CLO**.[5] Moreover, as in Dung's framework, stable extensions are also preferred.

Prop. 3 *Let* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, \mathbf{C}\rangle$ *be a GBAS and* $S \subseteq \mathbf{A}$*. If* S *is a stable extension of the GBAS then* S *is also a preferred extension of the GBAS.*

A thorough study of the generalized bipolar abstract argumentation framework would demand to define other semantics such as grounded one. However, this is not our purpose in this paper. We focus on the way to enforce different kinds of constraints related to necessary support.

4.2 A meta-framework encoding necessary support

The fact that "the acceptance of c is necessary to get the acceptance of b" can be encoded in another way. As explained in Section 3, the idea is to assume the existence of a special argument attacking b for which c is the *only* attacker. More precisely, if $c\mathbf{R}_{\text{sup}}b$, we create a new argument N_{cb} and two attacks $c\mathbf{R}_{\text{att}}N_{cb}$ and $N_{cb}\mathbf{R}_{\text{att}}b$. As c is the unique attacker of N_{cb}, "the acceptance of b implies the acceptance of c". The meaning of N_{cb} could be that the support from c to b is not active. A similar idea can be found in [28; 12] for the more general purpose of representing recursive and defeasible attacks and supports.

Def. 17 *Let* BAS $= \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}\rangle$ *with* $\mathbf{R}_{\text{sup}}$ *being a set of necessary supports. Let* $\mathbf{A}_n = \{N_{cb} | (c, b) \in \mathbf{R}_{\text{sup}}\}$ *and* $\mathbf{R}_n = \{(c, N_{cb}) | (c, b) \in \mathbf{R}_{\text{sup}}\} \cup \{(N_{cb}, b) | (c, b) \in \mathbf{R}_{\text{sup}}\}$*. The tuple* MAS $= \langle \mathbf{A} \cup \mathbf{A}_n, \mathbf{R}_{\text{att}} \cup \mathbf{R}_n\rangle$ *is the meta-argumentation system* [6] *associated with* BAS.

Let us check whether the minimal requirements are satisfied. Let us first consider constraint **TRA**. From $a\mathbf{R}_{\text{sup}}b$ and $b\mathbf{R}_{\text{sup}}c$, we obtain the sequence of attacks $a\mathbf{R}_{\text{att}}N_{ab}\mathbf{R}_{\text{att}}b\mathbf{R}_{\text{att}}N_{bc}\mathbf{R}_{\text{att}}c$. So, the acceptance of c implies the acceptance of b, which in turn implies the acceptance of a, as if we had directly encoded $a\mathbf{R}_{\text{sup}}c$. So, **TRA** is taken into account. The same result holds for **CLO**:

Prop. 4 *Let* BAS $= \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}\rangle$ *and its associated* MAS*. Let* $S \subseteq \mathbf{A} \cup \mathbf{A}_n$*. If* S *is admissible in* MAS*, then* $S \cap \mathbf{A}$ *is closed under* $\mathbf{R}_{\text{sup}}^{-1}$ *in* BAS.

Constraint **CFS** is not enforced. We only have the following property:

Prop. 5 *Let* BAS $= \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}\rangle$ *and its associated* MAS*. Let* a, b, c *be arguments of* $\mathbf{A}$*. If (*$a\mathbf{R}_{\text{att}}c$ *and* c `supports` b*) or (*$c\mathbf{R}_{\text{att}}b$ *and* c `supports` a*) then no admissible set in* MAS *contains* $\{a, b\}$.

Note that this result is weaker than **CFS** since it does not imply that $\{a, b\}$ is a conflicting set.

Obviously, stronger constraints such as **nATT** or **n+ATT** are not directly enforced. If $a\mathbf{R}_{\text{att}}c$ and $c\mathbf{R}_{\text{sup}}b$, we obtain the sequence $a\mathbf{R}_{\text{att}}c\mathbf{R}_{\text{att}}N_{cb}\mathbf{R}_{\text{att}}b$. No attack from a to b is added. However, we will see in Section 4.4 that the meta-argumentation framework associated with BAS enables to recover the extensions obtained when enforcing Constraint **nATT**.

[5] Note that enforcing coherence makes the set C useless due to Prop.2.

[6] Note that it is an argumentation system in Dung's sense.

4.3 A framework with complex attacks

In this subsection we discuss two frameworks enabling to handle necessary support through the addition of complex attacks. According to the various interpretations of the necessary support, all the complex attacks are not justified. For instance, if the necessary support models a subargument relation as in [26], only the secondary attack makes sense. Other works [22] have considered both cases of extended attack. However, to the best of our knowledge, there has been no formal study of the properties of these extended attacks, and of the consequences of these attacks on the acceptable sets of arguments.

From Def. 6, new attacks called `n+-attacks` can be generated inductively as follows:

Def. 18 (`n+-attacks`) *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *with* $\mathbf{R}_{\text{sup}}$ *being a set of necessary supports. There exists a* `n+-attack` *from a to b iff either $a\mathbf{R}_{\text{att}}b$, or there is a (case 1 or case 2) extended attack from a to b, or there exists an argument c s.t. a* `n+-attacks` *c and c* `supports` *b, or there exists an argument c s.t. c* `supports` *a and c* `n+-attacks` *b.* $\mathbf{N+}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}}$ *denoted the set of* `n+-attacks` *generated by* $\mathbf{R}_{\text{sup}}$ *on* $\mathbf{R}_{\text{att}}$*. The AS defined by* $\langle \mathbf{A}, \mathbf{N+}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}} \rangle$ *is denoted by* AS^{N^+}*.*

Obviously Constraints **TRA**, **nATT** and **n+ATT** are enforced in AS^{N^+}.

Let us now consider the case when the extended attacks are restricted to secondary attacks (Case 1 of extended attacks). Following the above definition, our purpose is to define a `n-attack` from a to b when either $a\mathbf{R}_{\text{att}}b$, or there exists a secondary attack from a to b, or there exists an argument c s.t. a `n-attacks` c and c `supports` b. Indeed, it is easy to prove that the formal definition of this `n-attack` can be simplified as follows:

Def. 19 (n-attack) *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$*. There is* `n-attack` *from a to b iff either $a\mathbf{R}_{\text{att}}b$, or there is a secondary attack from a to b.* $\mathbf{N}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}}$ *denoted the set of* `n-attacks` *generated by* $\mathbf{R}_{\text{sup}}$ *on* $\mathbf{R}_{\text{att}}$*. The AS defined by* $\langle \mathbf{A}, \mathbf{N}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}} \rangle$ *is denoted by* AS^{N}*.*

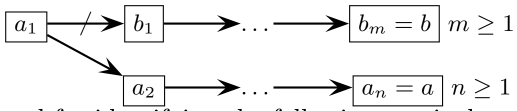
Note that both AS^{N} and AS^{N^+} are Dung's argumentation systems; so the classical notions given in Def. 2 and 3 can be applied without restriction, nor redefinition.

Obviously Constraints **TRA** and **nATT** are enforced in AS^{N}, whereas Constraint **n+ATT** is not.

Def. 18 looks complex. However the following proposition enables to rewrite `n+-attacks` and `n-attacks` in a form which will be much easier to handle for studying their properties.

Prop. 6 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$*. There is an* `n+-attack` *from a to b iff there is a sequence $a_1\mathbf{R}_{\text{att}}b_1\mathbf{R}_{\text{sup}}\ldots\mathbf{R}_{\text{sup}}b_m$, with $b_m = b$ and $m \geq 1$, and a sequence $a_1\mathbf{R}_{\text{sup}}\ldots\mathbf{R}_{\text{sup}}a_n$ with $a_n = a$ and $n \geq 1$.*

`n+-attacks` as defined by Prop. 6 can be illustrated by the following figure:



Moreover, Prop. 6 can be used for identifying the following particular cases:

– The case when $m = n = 1$ corresponds to a direct attack from a to b.
– The case when $n = 1$ and $m \geq 1$ corresponds to a `n-attack` from a to b (direct or secondary attacks, see Def. 19).
– The case when $n = 1$ and $m > 1$ corresponds to an extended attack - Case 1 (secondary attack) from a to b (see Def. 6).
– The case when $n > 1$ and $m = 1$ corresponds to an extended attack - Case 2 from a to b (see Def. 6).

An obvious consequence of this proposition is:

Corol. 1 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and its associated* AS^N *and* AS^{N^+}*. Let* $S \subseteq \mathbf{A}$*. If* S *is conflict-free in* AS^{N^+}*, then* S *is conflict-free in* AS^N*.*

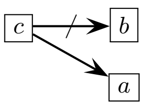
As said above, in some works necessary support can be handled by only considering `n-attacks`, that is by adding secondary attacks. However, although both cases of extended attacks are independent, we show that taking into account only `n-attacks` is already enough for inducing constraints on AS^{N^+}.

Prop. 7 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and its associated* AS^N*. If a* `n+-attack` *from* a *to* b *can be built from* BAS*, there exists no admissible set in* AS^N *containing* $\{a, b\}$*.*

As an immediate consequence (contrapositive of Prop. 7), we have:

Corol. 2 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and the associated* AS^N *and* AS^{N^+}*. Let* $S \subseteq \mathbf{A}$*. If* S *is admissible in* AS^N*, then* S *is conflict-free in* AS^{N^+}*.*

Ex. 4 *Consider* BAS *represented by the following graph:*



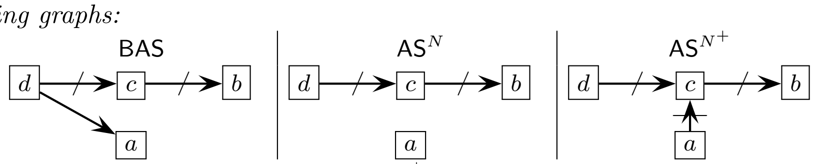
The associated AS^N *only contains the original attack from* c *to* b *(there is no secondary attack). If we consider only* `n-attacks`*, there is no conflict between* a *and* b*. However, it can be proved that no admissible set in* AS^N *contains* $\{a, b\}$*.*

The following results establish links between extensions in AS^N and AS^{N^+}.

Prop. 8 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and the associated* AS^N *and* AS^{N^+}*. Let* $S \subseteq \mathbf{A}$*. If* S *is admissible in* AS^N*, then* S *is also admissible in* AS^{N^+}*.*

The converse of Prop 8 generally does not hold as shown by the following example.

Ex. 5 *Consider* BAS *and its associated* AS^N *and* AS^{N^+} *represented by the following graphs:*



The set $\{a, b\}$ *is admissible in* AS^{N^+} *but is not admissible in* AS^N *(since* a *does not attack* c *in* AS^N*).*

However, the converse of Prop. 8 holds for maximal admissible sets:

Prop. 9 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and its associated* AS^N *and* AS^{N^+}*. Let* $S \subseteq \mathbf{A}$*.* S *is maximal admissible in* AS^{N^+} *iff* S *is maximal admissible in* AS^N*.*

The same holds for stable semantics:

Prop. 10 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}} \rangle$ *and its associated* AS^N *and* AS^{N^+}*. Let* $S \subseteq \mathbf{A}$*.* S *is stable in* AS^{N^+} *iff* S *is stable in* AS^N*.*

We conclude this section by providing results about the property of closure under the relation $\mathbf{R}_{\mathrm{sup}}^{-1}$.

Prop. 11 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}} \rangle$ *and its associated* AS^{N^+}*. Let* $S \subseteq \mathbf{A}$ *and* $a, b \in \mathbf{A}$*. If* S *is conflict-free in* AS^{N^+}*,* $a \in S$ *and* $b\mathbf{R}_{\mathrm{sup}}a$*, then* $S \cup \{b\}$ *is conflict-free in* AS^{N^+}*. If* S *is maximal (wrt* $\subseteq$*) conflict-free in* AS^{N^+}*, then* S *is closed for the relation* $\mathbf{R}_{\mathrm{sup}}^{-1}$*.*

Prop. 11 does not hold when considering AS^N instead of AS^{N^+}, as shown by the following example.

Ex.4 (cont'd) $S = \{a, b\}$ *is maximal conflict-free in* AS^N *but it is not closed under* $\mathbf{R}_{\mathrm{sup}}^{-1}$*. We have* $c\mathbf{R}_{\mathrm{sup}}a$ *but* $S \cup \{c\}$ *is not conflict-free in* AS^N*.*

However, the property of closure under $\mathbf{R}_{\mathrm{sup}}^{-1}$ is recovered in AS^N, if preferred (resp. stable) extensions are considered.

Prop. 12 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}} \rangle$ *and the associated* AS^N *and* AS^{N^+}*. Let* $S \subseteq \mathbf{A}$*. If* S *is a preferred extension in* AS^N *(resp.* AS^{N^+}*), then* S *is closed for the relation* $\mathbf{R}_{\mathrm{sup}}^{-1}$*. If* S *is stable in* AS^N *(resp.* AS^{N^+}*), then* S *is closed for the relation* $\mathbf{R}_{\mathrm{sup}}^{-1}$*.*

Due to Prop. 12, each stable (resp. preferred) extension of AS^N (resp. AS^{N^+}) is closed under $\mathbf{R}_{\mathrm{sup}}^{-1}$. In that sense, Constraint **CLO** is enforced in AS^N (resp. AS^{N^+}).

It remains to consider Constraint **CFS**. This constraint is obviously satisfied by AS^{N^+} since a new attack is built for each conflict in the sense of **CFS**, whereas the Dung's argumentation system AS^N does not capture all the conflicts induced by **CFS**, as illustrated by the following example.

Ex.3 (cont'd) *In the associated* AS^N*, there is one* `n-attacks` *from* x *to* c *and one from* x *to* b*.* $\{a, x\}$ *is the unique preferred extension of* AS^N*. It is also stable. Note that* $\{a, c\}$ *is conflict-free in* AS^N*. Nevertheless* $\{a, c\}$ *is a conflicting set in the sense of* **CFS***.*

4.4 Comparison between the different frameworks

In the previous sections, starting from a set of constraints, several frameworks (GBAS, MAS, AS^N and AS^{N^+}) have been proposed for handling necessary support. In this section, we compare these frameworks wrt two different points of view: the satisfaction of the constraints and the extensions that are produced.

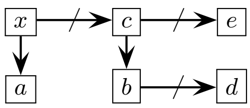
First, the following table synthesizes the previous results:

	GBAS	MAS	AS^N	AS^{N^+}
TRA	X	X	X	X
CLO	X	X	X	X
CFS	X	$-$	$-$	X
nATT	$-$	$-$	X	X
n+ATT	$-$	$-$	$-$	X

X (resp. $-$) means that the corresponding property is (resp. not) satisfied in the corresponding framework.

Now, let us consider AS^N and GBAS. We know that AS^N does not satisfy **CFS** whereas GBAS does. However, due to Prop. 7, if S is a conflicting set of GBAS, it is conflicting in AS^{N^+} and then there is no admissible set of AS^N containing S. Moreover, it can be proved that each preferred extension of GBAS is (generally strictly) included in a preferred extension of AS^N. This is illustrated by the following example.

Ex. 6 *Consider* BAS *represented by:*



In the associated GBAS*, we have* $\mathbf{C} = \{\{x, c\}, \{x, b\}, \{c, e\}, \{b, d\}, \{a, c\}, \{b, e\}\}$*. The unique preferred extension of* GBAS *is* $\{a, x, e\}$*. In the associated* AS^N*, the* `n-attacks` *from* x *to* b *is used for ensuring the acceptability of* d *wrt* $\{a, x, e\}$*. So, the unique preferred extension is* $\{a, d, x, e\}$*.*

Prop. 13 *Let* BAS $= \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and its associated* GBAS *and* AS^N*. Let* $S \subseteq \mathbf{A}$*. If* S *is admissible in* GBAS*, then* S *is also admissible in* AS^N*. If* S *is a preferred extension in* GBAS*, then* S *is included in a preferred extension of* AS^N*. If* S *is a stable extension in* GBAS*, then* S *is also a stable extension of* AS^N*.*

Note that Prop. 13 holds when considering AS^{N^+} instead of AS^N, due to Prop. 8, 9 and 10.

The next issue concerns the comparison between AS^N and the associated MAS of BAS. It seems that encoding a necessary support $c\mathbf{R}_{\text{sup}}b$ by a meta-argument N_{cb} and the sequence $a\mathbf{R}_{\text{att}}c\mathbf{R}_{\text{att}}N_{cb}\mathbf{R}_{\text{att}}b$ is less strong than encoding `n-attacks`. However, there is a correspondence between the extensions which are obtained in each framework.

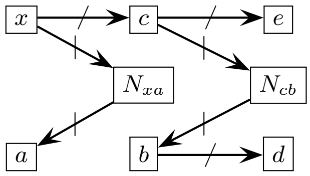
Prop. 14 *Let* BAS $= \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and its associated* MAS *and* AS^N*.*
- *Let* $S \subseteq \mathbf{A} \cup \mathbf{A}_n$*. If* S *is admissible in* MAS*, then* $S \cap \mathbf{A}$ *is also admissible in* AS^N*. If* S *is stable in* MAS*, then* $S \cap \mathbf{A}$ *is also stable in* AS^N*.*
- *Let* $S \subseteq \mathbf{A}$*. If* S *is a preferred extension in* AS^N*, there exists* S' *admissible in* MAS *such that* $S = S' \cap \mathbf{A}$*. If* S *is a stable extension in* AS^N*, then there exists* S' *stable in* MAS *such that* $S = S' \cap \mathbf{A}$*.*

From Prop. 13 and 14, the following comparison between GBAS and MAS can be easily established.

Prop. 15 *Let* BAS $= \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *and its associated* MAS *and* GBAS*. Let* $S \subseteq \mathbf{A}$*. If* S *is a preferred extension of* GBAS*, then there exists* S' *preferred in* MAS *such that* $S \subseteq S' \cap \mathbf{A}$*. If* S *is a stable extension of* GBAS*, then there exists* S' *stable in* MAS *such that* $S = S' \cap \mathbf{A}$*.*

The following example illustrates the above propositions.

Ex.6 (cont'd) *Consider the associated* MAS *represented by:*



In GBAS*, the unique preferred (and also stable) extension is the set* $\{a, x, e\}$*. In* AS^N*, the unique preferred (and also stable) extension is the set* $\{a, x, e, d\}$*. In* MAS*, the unique preferred (and also stable) extension is the set* $\{a, x, e, N_{cb}, d\}$*.*

5 Conclusion and future works

Recent studies in argumentation have addressed the notion of support, with several interpretations (such as deductive, evidential, necessary, backing) and several approaches developed independently. In this paper we focus on necessary support and show that the intended meaning of necessary support can induce different ways to handle it. Our main contribution is to propose an axiomatic approach that is helpful for understanding and comparing the different existing proposals for handling support. First, we have proposed different kinds of constraints that should be imposed to a bipolar argumentation system using necessary supports. Then we have studied different frameworks suitable for encoding these contraints.

This paper reports a preliminary work that could be pursued along different lines. First, our study must be deepened in order to give a more high-level analysis and comparison of all these frameworks. Then the axiomatic approach could be enriched by considering other constraints, such as for instance the strong requirement leading to the addition of symmetric attacks in the case of a necessary support. Moreover, it would be interesting to define such an axiomatic for other interpretations of support, or to consider other frameworks which do not explicitely define a notion of support, such as Abstract Dialectical Frameworks [6]. Another direction for further research would be to study how to encode necessary (or other variants) support by the addition of attacks of various strengths (see for instance [19; 7; 16; 17]). Moreover it would be interesting to see the link between our approaches and the ranking semantics proposed by [1].

References

1. Amgoud, L., Ben-Naim, J.: Ranking-based semantics for argumentation frameworks. In: Proc. of SUM, LNCS, vol. 8078, pp. 134–147. Springer (2013)
2. Amgoud, L., Cayrol, C.: A reasoning model based on the production of acceptable arguments. Annals of Mathematics and Artificial Intelligence 34, 197–216 (2002)
3. Amgoud, L., Cayrol, C., Lagasquie-Schiex, M.C., Livet, P.: On bipolarity in argumentation frameworks. Intl. Journal of Intelligent Systems 23, 1062–1093 (2008)
4. Amgoud, L., Maudet, N., Parsons, S.: Modelling dialogues using argumentation. In: Proc. of ICMAS. pp. 31–38 (2000)
5. Boella, G., Gabbay, D.M., van der Torre, L., Villata, S.: Support in abstract argumentation. In: Proc. of COMMA. pp. 111–122. IOS Press (2010)

6. Brewka, G., Woltran, S.: Abstract dialectical frameworks. In: Proc. of KR. pp. 102–111 (2010)
7. Cayrol, C., Devred, C., Lagasquie-Schiex, M.C.: Acceptability semantics accounting for strength of attacks in argumentation. In: Proc. of ECAI. pp. 995–996 (2010)
8. Cayrol, C., Lagasquie-Schiex, M.C.: On the acceptability of arguments in bipolar argumentation frameworks. In: Proc. of ECSQARU. pp. 378–389 (2005)
9. Cayrol, C., Lagasquie-Schiex, M.C.: Coalitions of arguments: a tool for handling bipolar argumentation frameworks. Intl. J. of Intelligent Systems 25, 83–109 (2010)
10. Cayrol, C., Lagasquie-Schiex, M.C.: Bipolarity in argumentation graphs: towards a better understanding. IJAR 54(7), 876–899 (2013)
11. Cayrol, C., Lagasquie-Schiex, M.C.: An axiomatic approach to support in argumentation. Tech. Rep. RR–2015-04–FR, IRIT (2015), http://www.irit.fr/publis/ADRIA/PapersMCL/Rapport-IRIT-2015-04.pdf
12. Cohen, A., Gottifredi, S., García, A.J., Simari, G.R.: An approach to abstract argumentation with recursive attack and support. Journal of Applied Logic (2014)
13. Cohen, A., Gottifredi, S., García, A.J., Simari, G.R.: A survey of different approaches to support in argumentation systems. The Knowledge Engineering Review 29, 513–550 (2014)
14. Coste-Marquis, S., Devred, C., Marquis, P.: Symmetric argumentation frameworks. In: Proceedings of ECSQARU, LNAI 3571. pp. 317–328. Springer-Verlag (2005)
15. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artificial Intelligence 77, 321–357 (1995)
16. Dunne, P., Hunter, A., McBurney, P., Parsons, S., Wooldridge, M.: Weighted argument systems: Basic definitions, algorithms, and complexity results. Artificial Intelligence 175(2), 457–486 (2011)
17. Kaci, S., Labreuche, C.: Arguing with valued preference relations. In: Proc. of ECSQARU. pp. 62–73 (2011)
18. Karacapilidis, N., Papadias, D.: Computer supported argumentation and collaborative decision making: the HERMES system. Information systems 26(4), 259–277 (2001)
19. Martinez, D.C., Garcia, A.J., Simari, G.R.: An abstract argumentation framework with varied-strength attacks. In: Proc of KR. pp. 135–143 (2008)
20. Nouioua, F.: AFs with necessities: further semantics and labelling characterization. In: Proc. of SUM (2013)
21. Nouioua, F., Risch, V.: Bipolar argumentation frameworks with specialized supports. In: Proc. of ICTAI. pp. 215–218. IEEE Computer Society (2010)
22. Nouioua, F., Risch, V.: Argumentation frameworks with necessities. In: Proc. of SUM. pp. 163–176. Springer-Verlag (2011)
23. Oren, N., Norman, T.J.: Semantics for evidence-based argumentation. In: Proc. of COMMA. pp. 276–284 (2008)
24. Oren, N., Reed, C., Luck, M.: Moving between argumentation frameworks. In: Proc. of COMMA. pp. 379–390. IOS Press (2010)
25. Polberg, S., Oren, N.: Revisiting support in abstract argumentation systems. In: Proc. of COMMA. pp. 369–376. IOS Press (2014)
26. Prakken, H.: On support relations in abstract argumentation as abstraction of inferential relations. In: Proc. of ECAI. pp. 735–740 (2014)
27. Verheij, B.: Deflog: on the logical interpretation of prima facie justified assumptions. Journal of Logic in Computation 13, 319–346 (2003)
28. Villata, S., Boella, G., Gabbay, D.M., van der Torre, L.: Modelling defeasible and prioritized support in bipolar argumentation. AMAI 66(1-4), 163–197 (2012)